But this formula does not enable us to obtain least values of p, q, and m, as n varies.

56. Proposed by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.

If $\phi(R)$ is the number of integers which are less than R and prime to it, and if y is prime to R, show that $y^{\Phi(R)}-1\equiv 0(\text{mod}.R)$.

Solution by the PROPOSER, and J. O. MAHONEY, B. E., M. Sc., Lynnville, Tenn.

Let $1, m, n, p, \ldots\ldots(R-1)$ denote the $\phi(R)$ numbers less than R and prime to it ; now y can be any one of those numbers.

$\therefore y, my, ny, py, \ldots\ldots(R-1)y$ are all prime to R and all different.

There are $\phi(R)$ of such products and since when these products are divided by R the remainders are all prime to R and all different, the $\phi(R)$ remainders must be $1, m, n, p, \ldots\ldots(R-1)$ though not necessarily in this order.

$\therefore y.my.ny.py\ldots\ldots(R-1)y$ must differ from $1.m.n.p\ldots\ldots(R-1)$ by a multiple of R.

$\therefore \{y^{\Phi(R)}-1\}mnp\ldots\ldots(R-1)=$a multiple of R.

But $mnp\ldots\ldots(R-1)$ is prime to R.

$\therefore y^{\Phi(R)}-1\equiv 0(\text{mod}.R)$.

57. Proposed by JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

Each of *five* of the digits may be the terminal figure of a perfect integral square. Each of *eighteen* combinations of two digits may be the *two* terminal figures of an integral square. Each of *one hundred and nineteen* combinations of three digits may be the *three* terminal figures of an integral square. *Under these conditions*, what is the greatest number of arrangements of the nine digits, all taken together, whose three terminal figures shall be those of a square number ?

No solution of this difficult problem has been received. Can any of our readers furnish the desired solution ? EDITOR.

MISCELLANEOUS.

53. Proposed by J. A. CALDERHEAD, M. Sc., Professor of Mathematics in Curry University, Pittsburg, Pa.

(a) What is the highest north latitude in which the Sun will shine in at the north window of a building at least once in a year ?

(b) How many days will it shine in at the north window of a building in latitude 41° N. ?

Note by SAMUEL HART WRIGHT, M. D., A. M., Ph. D., Penn Yan, N. Y.

Whenever the Sun, or any part of it, is north of the prime vertical, it must then shine on the north side of buildings. From the time of vernal equinox, to the autumnal equinox, the Sun will be north of the prime vertical during some part of every day, and will shine on the north side of buildings some part of *every* day for about half a year, and in *all* latitudes north of the equator. Hence the answer for (a) is 90° N. latitude, and for (b) 186 days, but if the Sun's upper

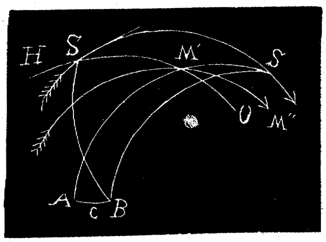
limb, and refraction, be considered the days will be 187 or 188. The answer for 41° N. is *good* for *any* other latitude north, while the problem seems to imply that an answer for 41° is different for other latitudes.

54. Proposed by S. HART WRIGHT, M. D., A. M., Ph. D., Penn Yan, N. Y.

On latitude 40° N.$=\lambda$, when the Moon's declination is 5° 23′ N.$=\delta$, and the Sun's 9° 52′ S.$=-\delta$, how long after sunset will the two horns or cusps of the Moon's crescent (recently new) set at the same moment, the crescent with its back *down* having touched the horizon first? Semi-diameters, refraction, and parallax not considered.

I. Solution by the PROPOSER.

Let B be the celestial north pole, A the zenith, AB an arc of the meridian equal the co-latitude$=c=50°$, HO a portion of the horizon, SS' and MM'' portions of the diurnal arcs of the Sun and Moon, the Sun setting at S, and the Moon at M'; BS=the polar distance of the Sun$=BS'$, and BM' the polar distance of the Moon, and AM' the zenith distance of the Moon$=90°$.



Produce the vertical circle AM' to S', S' being the place of the Sun when the Moon sets at M'. The line joining the Moon's cusps must be at right angles to the line $M'S'$ joining the centers of the Sun and Moon, and as the horison is at right angles to $AM'S'$, the line of the cusps must lie on the horizon and set when the Moon's center sets. Put $\angle ABS=\phi$=Sun's hour angle when it sets, and $\angle ABS'=\theta$= Sun's hour angle when the Moon sets, and $\angle ABM'=\psi$=Moon's hour angle when it sets.

Then we have $\cos\phi=\tan\delta'\tan\lambda$. $\therefore$ $\phi=81° \ 36' \ 29''$, and $\cos\psi=-\tan\delta\tan\lambda$. $\therefore$ $\psi=94° \ 32' \ 7''$. Take an auxiliary arc χ', and $\tan\chi'=\cos\psi\cot\delta$. $\therefore$ $\chi'=40° \ 0' \ 1''$, then $\cot A=\sin(c-\chi')\cot\psi\operatorname{cosec}\chi'$. $\therefore$ $A=82° \ 57' \ 55''$. Take an auxiliary angle γ', and $\cot\gamma'=\tan A\sin\lambda$. $\therefore$ $\gamma'=10° \ 52' \ 2''$. Then $\cos\gamma'\cot\lambda\tan-\delta'=-\cos\gamma$. $\therefore$ $\gamma=101° \ 44' \ 43''$, and $\angle ABS'=\gamma'+\gamma=\theta=112° \ 36' \ 45''$, and $\theta-\phi=31° \ 0' \ 16''$=2 hours, 4 minutes, 1 second.

NOTE. The synchronous setting or rising of the cusps of a crescent Moon, is a phenomenon which must occur frequently in the tropics, and rarely or not at all beyond latitude 45°. On the 4th of July, 1897, such a moonset was very nearly accomplished, and another, almost perfect, will occur February 22, 1898, the declinations being then as given in the problem. Few persons in the northern states have ever seen the Moon set with both horns vertical.

II. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

Let O be the observer, Z his zenith, HMK Moon's path, $GCSL$ Sun's path, $TEFR$ celestial equator, $AMCB$ the horizon. Let M be the position of the Moon when setting. Then, in order that the horns may set at the same time, S, M, where S is the Sun, must be on the same meridian, $ZMSN$.

$AP=\lambda=40°$. $ME=\delta=5° \ 23'$ N. $SF=\delta_1=9° \ 52'$ S. In the triangle